Demand Routing for Intermodal Transportation Networks using a Design-as-Inference Approach

Lei Cao, Paul M. Goggans, R. Wesley Henderson

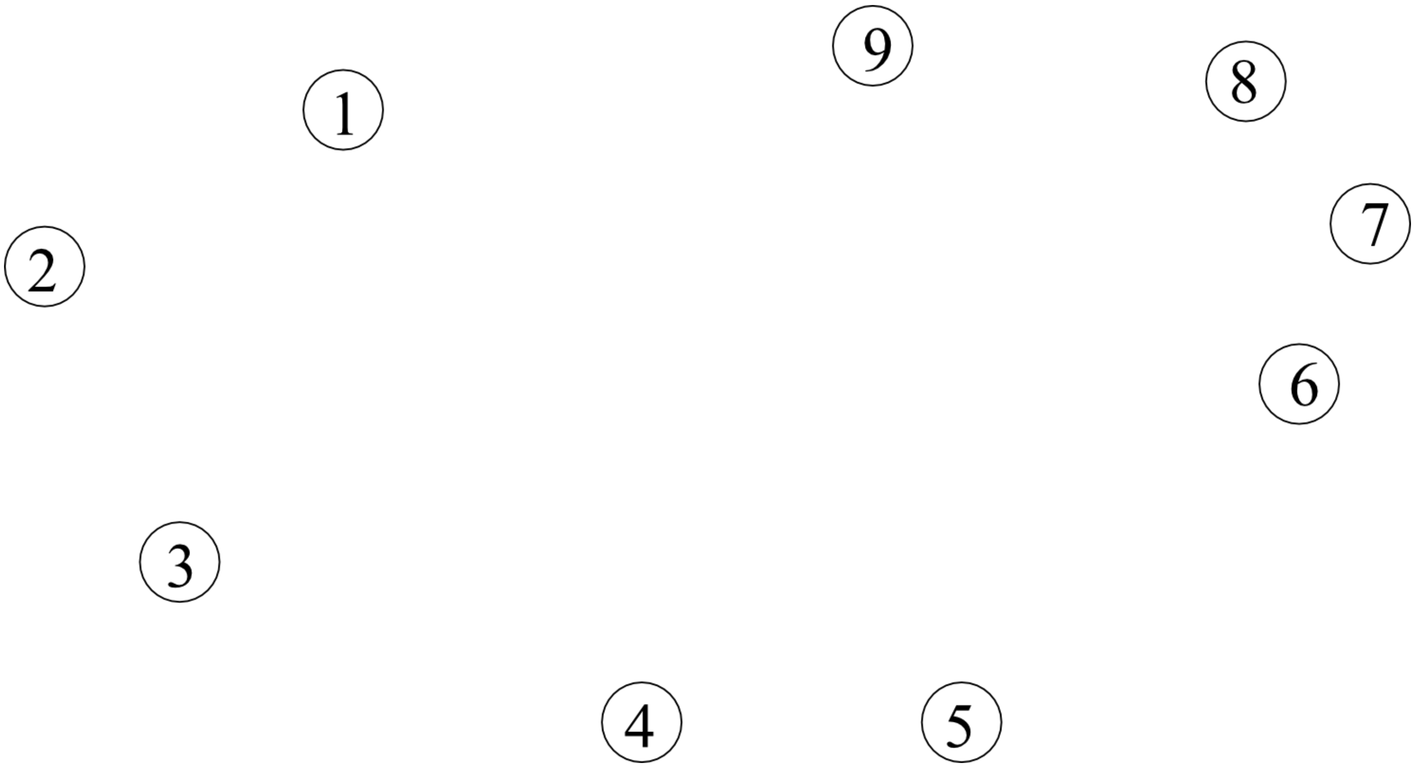
Dept. of Electrical Engineering
University of Mississippi

Orlando, FL – April 4, 2013

Outline

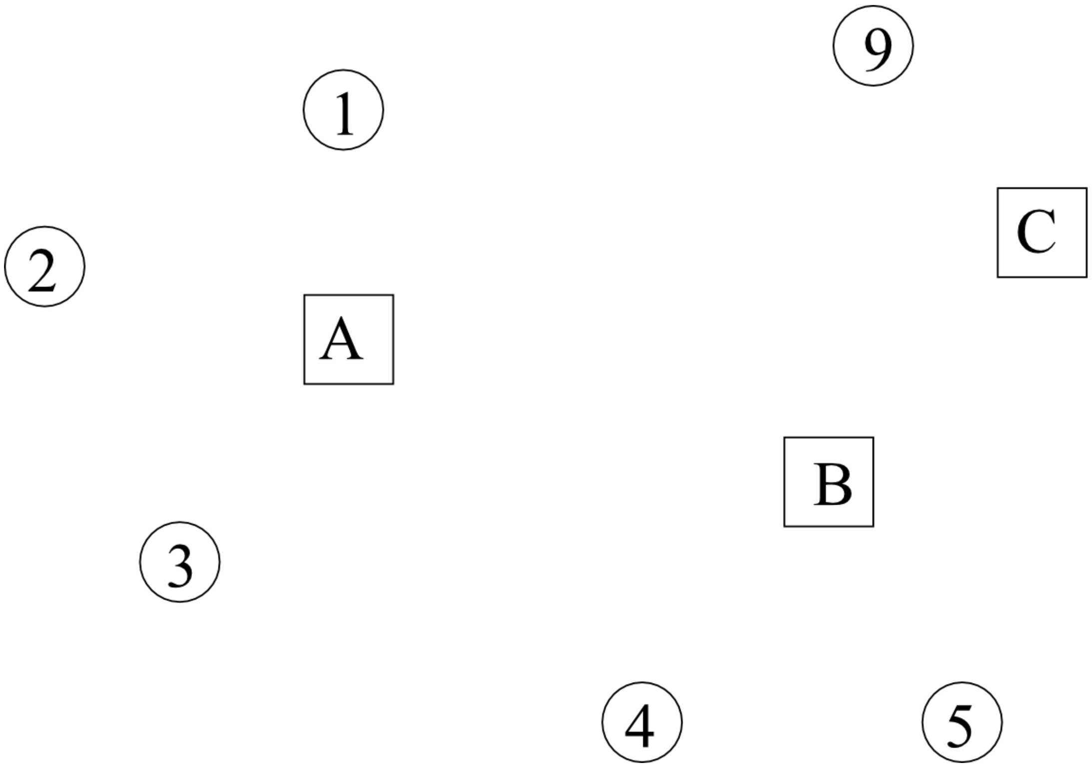
- Problem
  - Intermodal freight routing
  - Terminal planning
- Methods
  - Design-as-inference
  - Markov-chain Monte Carlo (MCMC)
- Current status & future work

Intermodal Freight Network



$$N = 9 \times 8 \quad \mathbf{d} = \{d_1, d_2, ..., d_N\}$$

Intermodal Freight Network

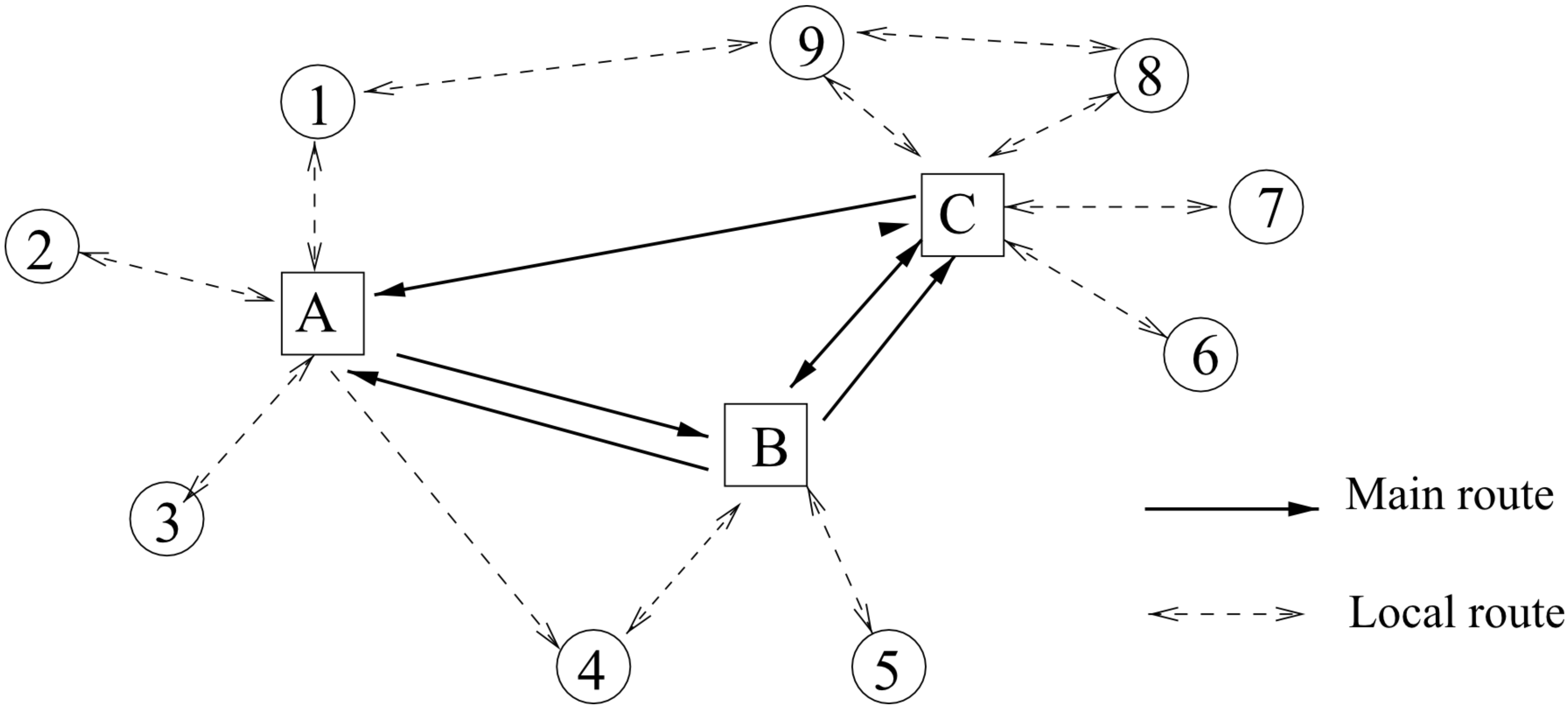


$$\mathbf{k} = \{k_1, k_2, ..., k_K\}$$

$$\mathbf{T} = \{T_1, T_2, ..., T_K\}$$

$$K = 3 \quad M = 3 \times 2$$

Intermodal Freight Network



$$\mathbf{x} = \{x_1^0, x_1^1, ..., x_1^M, ..., x_N^0, x_N^1, ..., x_N^M\}$$

$$\mathbf{c} = \{c_1^0, c_1^1, ..., c_1^M, ..., c_N^0, c_N^1, ..., c_N^M\}$$

Freight Demand Routing

- Routing demand through fixed network
- Must meet demand estimates and terminal capacity constraints

Terminal Planning

- Terminals may be added to ease congestion, add capacity, and ultimately reduce the cost of using the network
- Costs of building and running terminal must be weighed against the reduction in transportation cost by routing a greater portion of demand over rail
- Fixed cost of terminal becomes important

Cost and Terminal Capacity

$$C(\mathbf{x}) = \sum_{n=1}^{N} \sum_{m=0}^{M} c_n^m x_n^m + \sum_{k=1}^{K} F_K$$

$$t_k(\mathbf{x}) = \sum_{n=1}^{N} \sum_{m \in M^k} x_n^m$$

$$t_k(\mathbf{x}) \leq T_k \text{ for } 1 \leq k \leq K$$

Design-as-Inference

$$d_n = \sum_{m=0}^{M} x_n^m \qquad \text{for } 1 \leq n \leq N$$

- Demand estimates can be distributions
- Enables incorporation of uncertainty in demand estimates

Design-as-Inference

$$\underbrace{p(\mathbf{x}|\mathbf{d},\mathbf{c},\mathbf{T},\mathbf{k})}_{\substack{\text{Posterior}\\ \mathrm{P}(\mathbf{x})}} \propto \underbrace{p(\mathbf{x}|\mathbf{T},\mathbf{k},\mathbf{d})}_{\substack{\text{Prior}\\ \pi(\mathbf{x})}} \underbrace{p(\mathbf{c}|\mathbf{x},\mathbf{k})}_{\substack{\text{Likelihood}\\ \mathrm{L}(\mathbf{x})}}$$

$$\log \mathrm{L}(\mathbf{x}) = -\frac{1}{2\sigma_c^2} C(\mathbf{x})$$

$$p(\mathbf{k}|\mathbf{d},\mathbf{c},\mathbf{T}) \propto Z$$

Design-as-Inference

$$Z = \int \pi(\mathbf{x}) \mathrm{L}(\mathbf{x}) \, \mathrm{d}\mathbf{x}$$

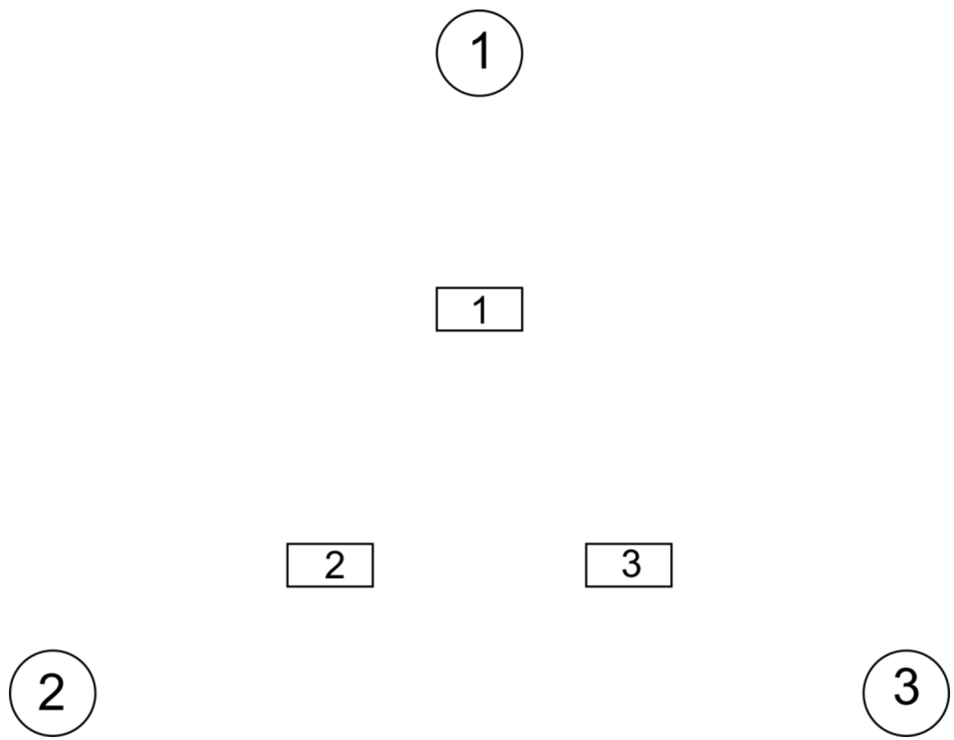
- Evidence is used to compare different proposed terminal locations
- Terminals that would be a net benefit to the network have a higher evidence value

MCMC

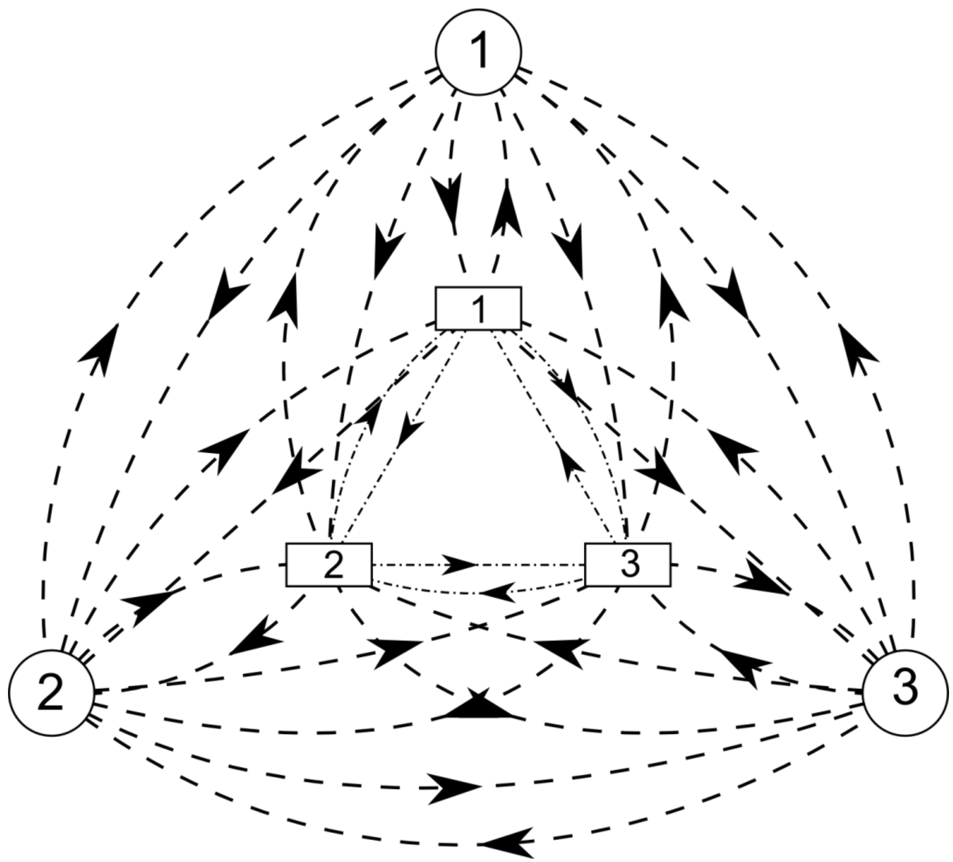
$$\xi(\mathcal{L}) = \int_{\mathrm{L}(\mathbf{x})>\mathcal{L}} \pi(\mathbf{x})\,\mathrm{d}\mathbf{x}$$

$$Z = \int_0^1 \mathcal{L}(\xi)\,\mathrm{d}\xi$$

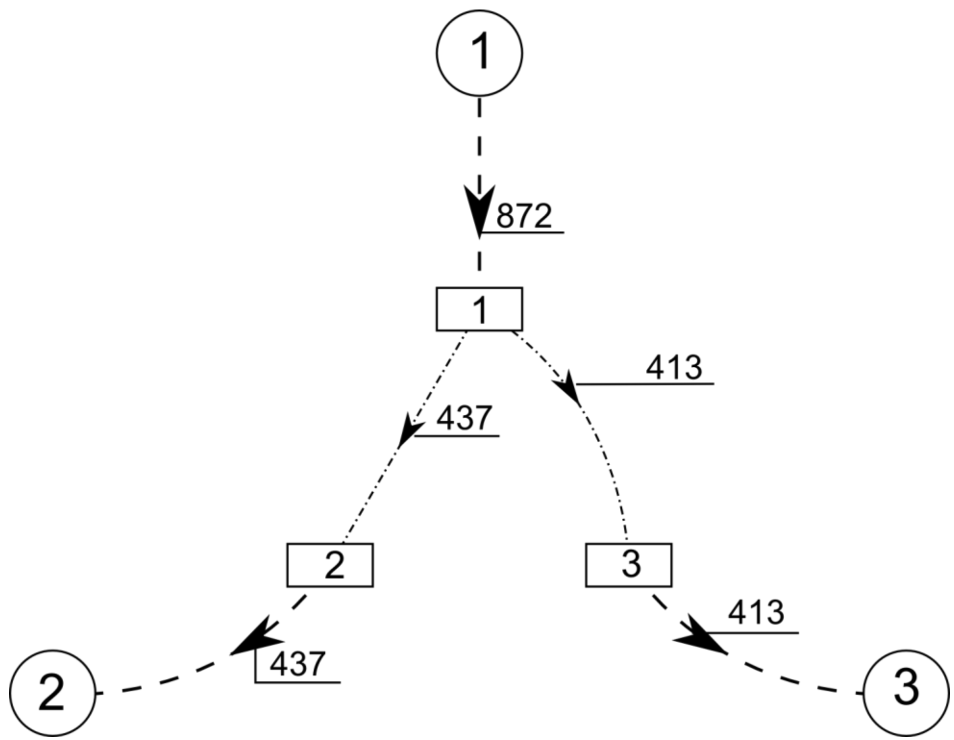
Preliminary Results



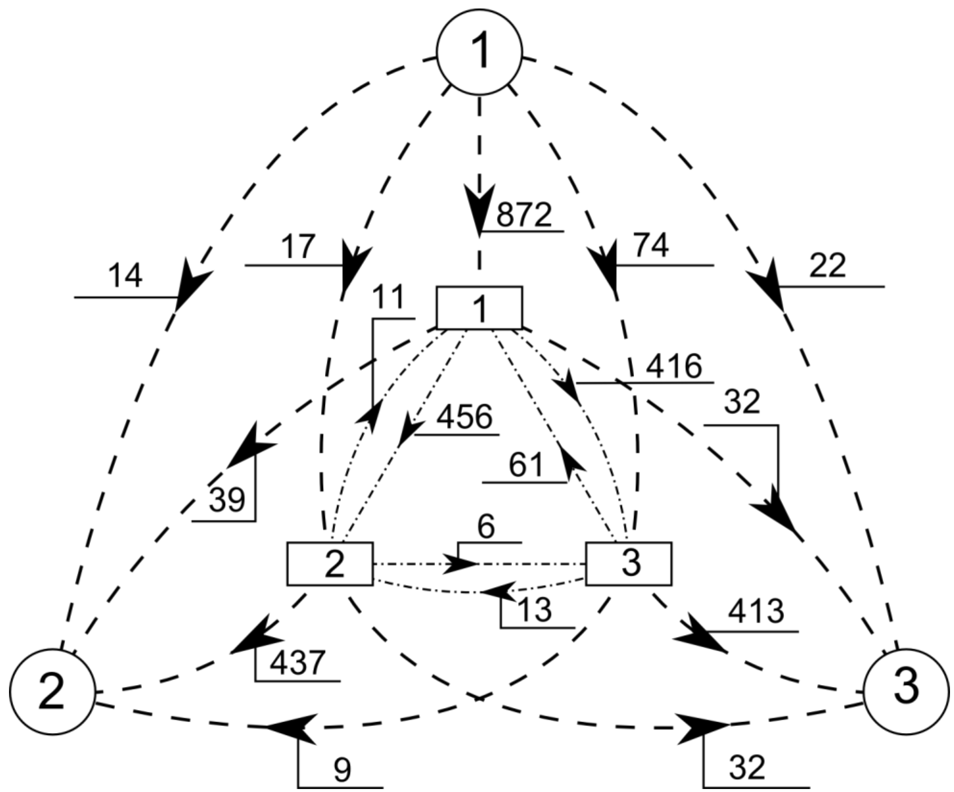
Preliminary Results



Preliminary Results



Preliminary Results



Roadmap

- Address terminal addition
- Improve prior exploration in nested sampling
- Use mapping data to generate more realistic test cases

Acknowledgement

- Thanks to NCITEC and USDOT for funding this project